P425/2 APPLIED MATHEMATICS

POST MOCK ANALYSIS-2017

1. STATISTICS

1. The ages and systolic pressure of 12 women working in a organization were recorded as shown below.

Age	56	42	72	36	63	42	68	60	47	55	49	72
Blood pressure	147	125	160	118	149	140	152	155	128	150	145	160

(a) Plot a scatter diagram and draw the line of best fit.

(b) Estimate the blood pressure for a woman aged 45 years.

(b) Calculate the rank correlation coefficient for the data and comment on your result.

2. The data below shows the ages of some students in a certain college given to the nearest year.

Age in years	17-19	20-22	23-25	26-28	29-31
No. of students	3	7	13	25	12

(a) On a graph paper, draw a cumulative frequency curve for the data

(b) Use the graph in (a)above to determine the;

(i) median age

(ii) 80^{th} percentile age

(iii) percentage of students whose ages lie in the range 24 years to 30 years.

3. At the end of the Big league, the goals scored by the team forming the league with their positions are shown below:

Position	1	2	3	4	5	6	7	8	9	10
Number of goals scored	37	19	34	28	26	20	21	18	19	16

Calculate the rank correlation between the position and the goals scored. Comment on your results.

4.The table below shows the distribution of the weights of a random sample of mango seedlings.

Weight(gms)	<10	<15	<30	<45	<55	<65
Frequency	25	30	30	18	30	12

(a) Calculate the mean and standard deviation.

(b) Construct a histogram and use it to estimate the modal value of the weight.

5.The table below shows the time taken to load vehicles in a day.

Time (minutes)	Number of Vehicle
40-45	4
45-50	13
50-55	17
55-60	44
60-70	59
70-80	7

(a) Draw a histogram and use it to estimate the modal time.

(b) Find the: (i) Mean time (ii) Median time (iii) Standard deviation of the time

6. The mock examination and average final examination marks are given in the table below:

Mock marks	28	34	36	42	48	52	54	60
Average final marks	54	62	68	70	76	66	76	74

Calculate the rank correlation coefficient between the marks and comment on your result.

7. The following table summarizes the distance to the nearest mile, travelled to work by a random sample of commuters

Distance (miles)	0-9	10-19	20-29	30-39	40-49	50-59
No. of commuters	15	38	22	15	8	2

a) Calculate the:

i) Mean

ii) Median

iii) Standard deviation

(b) Draw a histogram to represent the data and use it to estimate the mode

8. The marks of 40 students in a test were as follows:

Marks	30 –	40 –	50 –	60 –	70 –	80 –
Frequency	8	5	12	9	6	0

Calculate the; (i) mode. (ii) mean mark.

9. The marks of 100 students were as follows:

Marks	Frequency
Below 5	6
5 and under 10	11
10 and under 15	17
15 and under 25	28
25 and under 30	20
30 and under 35	15

35 and under 40	3

(a) Calculate the:

(i) upper quartile mark.

(ii) variance of the distribution.

(b) Display the data on a histogram and use it to estimate the mode.

10. The table below shows the height (cm) and the age (years) of a group of students.

Height (cm)	120	152	90	165	152	144
Age (years)	16	17	10	17	14	15

Calculate the rank correlation coefficient between height and age and comment on the relationship.

11. The table below shows the weight of seeds of a certain type of plant.

Weight (grams)	< 0.10	< 0.25	< 0.35	< 0.50	< 0.60	<0.65	< 0.80
Frequency	2	3	5	9	3	2	3

(a) Calculate the (i) mean weight, (ii) standard deviation weight.

Draw a histogram and use it to estimate the modal weight.

12. The following grades were obtained by 8 candidates in Mathematics and General paper.

Mathematics	A	O	B	F	E	C	D	B
General Paper	C_3	D_2	D_1	P_8	P_8	D_2	C_3	D_2

(a) Calculate the rank correlation coefficient for the grades.

(b) Comment on your result at 1% level of significance.

13. The table below shows the weights (W) in kg of 150 patients who visited a certain health unit during a certain week.

Weight (kg)	Number of patients
$0 \leq W \leq 19$	30
$20 \leq W \leq 29$	46
$30 \leq W \leq 39$	70
$40 \leq W \leq 49$	102
$50 \leq W \leq 59$	130
$60 \leq W \leq 69$	142
$70 \leq W \leq 79$	150

(a) Draw a cumulative frequency curve and use it to estimate the number of patients who weigh between 13kg and 53kg who visited the health unit.

(b) Calculate the; (i) mean weight (ii) modal weight.

14. The table below shows the values of quantities x and y

x	51	62	64	47	54	44	52	68	61	56
y	45	54	58	46	49	43	48	59	56	53

(a) Plot a scatter diagram and draw the line of best fit.

(b) Find the value of x when the value of y is 55

(c) Calculate the rank correlation coefficient and comment on your result.

15. The table below shows the marks scored in a mathematics test by some students:

Marks	30-34	35-39	40-44	45-49	50-54	55-59	60-64	65-69	70-74	75-79
frequency	3	2	5	8	15	17	13	7	6	4

(a) Calculate; (i) the standard deviation (ii) the mode

(b) Construct an Ogive and use it to obtain the 77^{th} percentile

16.The following table summarizes the distance to the nearest mile travelled to work by a random sample of commuters.

Distance (miles)	Number of commuters
0-9	15
10-19	38
20-29	22
30-39	15
40-49	8
50-59	2

(a) Estimate the; (i) mean (ii) standard deviation of this distribution

(b)Find the 98.5% confidence interval for the mean distance travelled by the commuters.

17. The marks of 6 students in French and Biology were as follows:

French	90	60	80	54	86	70
Biology	48	75	60	78	65	50

(a) Plot a scatter graph and draw the line of best fit.

(b) Find the marks of French of a student who got 55 marks in Biology.

(c) Calculate a rank correlation coefficient for the scores in the two tests. Comment on your result at 5% level of significance.

18. The length (h) in inches of 40 nails were as follows:

Length (h)	Frequency
$3.0 \leq W < 3.5$	8
$3.5 \leq W < 4.0$	5
$4.0 \leq W < 5.5$	12
$5.5 \leq W < 6.0$	9
$6.0 \leq W < 6.5$	6

(a) Display the data on a histogram and use it to estimate the mode

(b) Calculate the standard deviation of the distribution.

19. The table below shows the marks awarded to eight competitors by two judges in a music competition.

	A	B	C	D	E	F	G	H
Judge I	63	65	55	63	63	55	60	58
Judge II	77	70	60	75	68	58	54	68

(a) Calculate the rank correlation coefficient between the two judges.

(b) Comment on the relationship between the two Judges at 5% level of significance.

20. A train moving between two cities departs every day at 8:28am. The table below shows the distribution of time taken for the train to travel between the cities.

Time (min)	Frequency
80-<85	6
85-<90	12
90-<95	22
95-<100	31
100-<105	15
105-<110	7
110-<115	4
115-<120	2
120-<125	5

(a) Calculate the average time of the train.

(b) Draw a cumulative frequency curve and use it to estimate the median time of arrival.

End of statistics

2. PROBABILITY

1. At a certain supermarket, consumers only pay by cach or use credit cards. The ratio of customers who pay by cash to those who pay by credit cards is 3:2. If a random sample of 10 customers is selected, calculate:

(i) the expected number of customers who pay by cash.

(ii) the probability that exactly two customers pay by cash.

2. The probability that two independent events occur together is $\frac{2}{15}$. The probability that either one of the events or both of them occur is $\frac{3}{5}$. Find the individual probabilities of the two events.

3. X is a uniform continuous random variable such that

$$P(X \leq x) = \begin{cases} 0\,; x \leq a \\ \frac{x-2}{4}; a \leq x \leq b \\ 1; x \geq b \end{cases}$$

(i) Find the value of the constant a and b

(ii) Calculate E(X) and Var (X)

(iii) Sketch the graphs of the p.d.f and c.d.f of X.

4. The weight of bars of soap produced in a certain factory are normally distributed.
(a). Given that 1% of the bars weigh less than 140 g and 2% weigh more than 165 g, find the mean and variance of the distribution.
(b). If the variance in (a) above is reduced by 30%, find the proportion of bars that would be expected to weigh less than 145 g.

5. Two events X and Y are such that P(Y) = 0.7, P(X) = 0.4 and P(Y/X) =0.3. Find the probability that neither X nor Y occurs

6. A box of chalk has 12 pieces of which 3 are broken. If a random sample of 40 such boxes is taken, find the probability that between 9 and 13 boxes have broken pieces of chalk

7. A random variable Y has a distribution function given by

$$f(y) = \begin{cases} \frac{y}{k}; & y = 1, 2, 3, \ldots, n \\ 0\,; otherwise \end{cases}$$

If the mean of Y is 3, find;
(a) the values of n and k
(b) the variance of Y
(c) $P(Y = 2/Y < 3)$

8. Box C contains 7 yellow and 3 green oranges. Box B contains 4 yellow and 5 green oranges. An orange is randomly selected from A, and placed in B and then two oranges are randomly selected from B, one at a time without replacement, and put in box A,
(i) Calculate the probability that the oranges transferred from A to B and back to A are of the same colour.
(b) Draw a probability distribution table for the number of green oranges in box A after the experiment.
(c) Find the mean and standard deviation of the number of oranges in box A after the experiment.

9. 40 percent of the students, in a certain school own personal computers. If 24 students are selected at random, find the probability that between 8 and 15 students own personal computers.

10. A random sample of 100 Adults is taken from a population. The sample is found to have a mean weight of 76.0 and variance of 144.00. Find the 97% confidence limits for the mean of the population.

11. The height of pupils in a school is normally distributed with mean 130 cm and standard deviation of 7 cm. If a pupil is selected at random. Find the probability that his/her height is
(i) above 144 cm
(ii) less than 123 cm
(b) determine the lower and upper quartile heights

(c) If 4000 pupils are selected at random, determine the number whose height will lie between126.5 cm and 137 cm.

12. A continuous random variable X has a probability density function given by;

$$f(x) = \begin{cases} 0.5kx^2 \,; 0 \le x \le 2 \\ k(4-x); 2 \le x \le 4 \\ 0; \quad otherwise \end{cases}$$

(a) Sketch the gradient of f(x)
(b) Determine the value of k
(c) Find the (i)Mean (ii) Median.

13. Two hunters A and B shoot at an animal. The probability that A hits the animal is ½ and the probability that B does not hit the animal is 1/3. A shoots at the animal first, and then B. find the probability that:
(i) Both A and B hit the animal
(ii) only one hits the animal

14. A machine manufacturing nails make approximately 85% that are within the set tolerance limits. If a random sample of 200 nails is taken, find the probability that more than 21 nails will be outside the tolerance limits.

15. A continuous random variable x is defined by the p.d.f

$$f(x) = \begin{cases} \frac{1}{5}(x+2), & -2 < x < 0 \\ \frac{2}{15}(3-x), & 0 < x < 3 \\ 0 \quad , & elsewhere \end{cases}$$

a) sketch f(x)

b) Determine the i) expectation of x

ii) Cumulative distribution function, F(x) hence find $P(-1 \le x < 1)$

16. Strips of a metal are cut to length (cm) which follows a normal distribution with mean 50 cm and variance 0.25cm^2. find the;
i) probability that the length selected at random will lie between 49.25cm and 50.75cm.

ii) length range of the middle 70% of the cut of the strips

17. The distribution function of a continuous random variable X is as follows:

$$F(x)=\begin{cases}0 & , \quad x<1\\ \frac{1}{4}(x-1)^2 & , \quad 1\le x\le 3\\ 1 & , \quad x>3\end{cases}$$

Find: (i) $P(1.5\le X\le 2)$. (ii) The p.d.f of X .

18. Joan can either use a bus or a taxi to go to work. The chance that she uses a bus is $\frac{1}{4}$. If she uses a bus, the chance that she will be late is $\frac{2}{3}$, otherwise it is $\frac{3}{5}$. Find the chance that she will:

(i) be late on a particular day.

(ii) use a taxi given that she is punctual

19. (a)Box P contains 3 white and 4 blue beads while box Q contains 5 white and 3 blue beads. A bead is drawn at random from P and put into Q and then a bead is taken from Q and put into P. Find the probability that the bead drawn now from box P is white.

(b) Bag X contains 4 red and 3 blue pens, while bag Y contains 3 red and 2 blue pens. A bag is selected at random and two pens are drawn from it without replacement. Find the probability of picking:

(i) pens of different colours.

(ii) bag Y given that the pens drawn are of the same colour.

20. (a)The weights of 400 books are normally distributed with mean 50 g and standard deviation 2 g. Find how many books weigh less than 52.48 g.

(b) A random sample of 100 items taken from a normal population had the following data: $\overline{X} = 82$ and $\sum X^2 = 686800$. Find the:

(i) unbiased estimate of the population standard deviation.

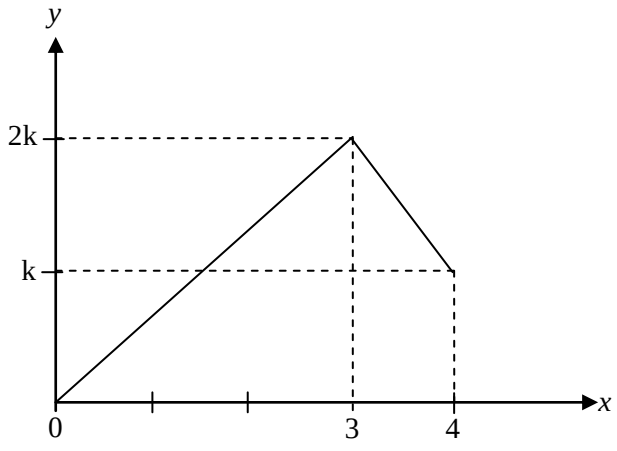
(ii) 97% confidence interval for the population mean.

21. Box A contains 4 yellow and 3 green oranges while box B contains 5 yellow and 7 green oranges. An orange is randomly selected from A and placed in B and then an orange is randomly selected from B and put back in A. If X is the number of yellow oranges remaining in box A after the operation,

(i) draw a probability distribution table for X and show that X is a random variable,

(ii) find the mean of X.

22. The probability density function of a continuous random variable X is represented graphically as shown.



Find the

(i) value of k,

(ii) equations of the p.d.f. f(x),

(iii) P(X>2/X<3.5).

23. The ages of students attending a certain function are normally distributed with mean 18 years. If 85.1% of the students have their ages greater than 17 years, find the probability that

(a) a student selected at random has an age between 16 and 19 years.

(b) a random sample of 16 students have their mean ages between 17.5 and 18.7 years.

24. Events A and B are such that $P(A) = \frac{8}{15}, P(B) = \frac{1}{3}$ and $P\left(\frac{A}{B}\right) = \frac{1}{5}$.

Find the probability that;

(i) neither A nor B occurs

(ii) Event B does not happen if either event A has occurred.

25. A discrete random variable X has a cumulative distribution function (c.d.f) given below.

x	1	2	3
F(x)	λ	4λ	9λ

Find the;

(a) value of λ

(b) mean of X.

26. A continuous random variable X has the probability density function given below

$$f(x) = \begin{cases} \frac{x^2}{27}; & 0 \leq x \leq \beta \\ \frac{1}{3}; & \beta \leq x \leq \alpha \\ 0; elsewhere \end{cases}$$

where α and β are constants

(a) Find the values of α and β, hence sketch $f(x)$

(b) Calculate the;

(i) median of X

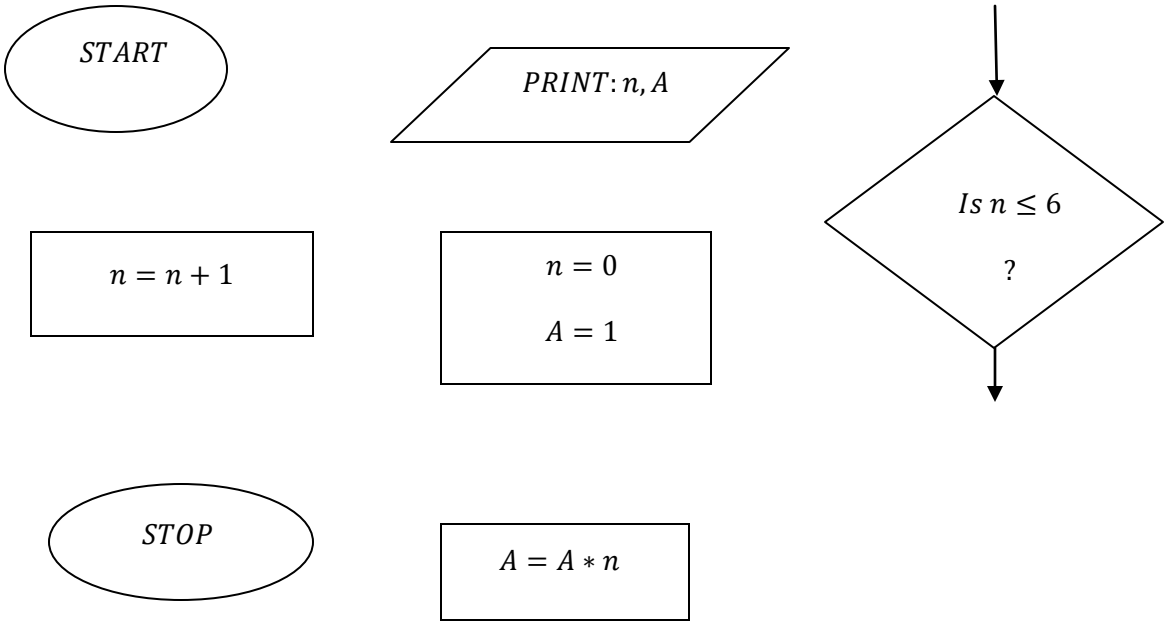
(ii) Mean of X.

End of Probability

3. NUMERICAL METHODS

1. Determine the range of values in which $\frac{3.02\times1.2}{6.1} - 1.2$ lies. Hence find the maximum possible error in the expression.

2. Given below are points of a flow chart not arranged in order.



Rearrange them and draw a complete logical flow chart.

3. (a). Use the trapezium rule with 6 ordinates to estimate the area of $y = \int_1^2 xe^{-2x}dx$
Give your answer correct to **three** significant figures.
(b). (i). Find the exact value of $\int_1^2 xe^{-2x}dx$, correct to **three** significant figures.
(ii). Calculate the percentage error in your estimation in (a) above.
(iii). Suggest how the percentage error may be reduced.

4. (a). (i). On the same axes, draw graphs of $y = x^2$ and $y = \cos x$ for $0 \leq x \leq \frac{\pi}{2}$
at intervals of $\frac{\pi}{8}$.

(ii). From your graphs, obtain to **one** decimal place, an approximate root of the equation $x^2 - \cos x = 0$.
(b). Using Newton-Raphson method, find the root of the equation $x^2 - \cos x = 0$, taking the approximate root in (a) as an initial approximation. Give your answer correct to **three** decimal places.

4. The table below shows values of $\sqrt{x}$ for given values of x:

x	0.12	0.13	0.14	0.15
$\sqrt{x}$	0.3464	0.3606	0.3742	0.3873

Using linear interpolation or extrapolation, find
(a) $\sqrt{0.135}$
(b) The value of x corresponding to $\sqrt{x} = 0.403$

5. (a) Derive the iterative formula based on Newton- Raphson process for obtaining the reciprocal of a number N.
(b) Hence, using the Newton Raphson process once, and taking $x_0 = 0.1$, find the approximate value of $\frac{1}{11}$ correct to 2 significant figures.

6. (a) Given that a and b are estimated with corresponding errors. ΔA and ΔBShow that the relative error in the product ab is: $\left|\frac{\Delta A}{a}\right| + \left|\frac{\Delta B}{b}\right|$
(b) The values p = 4.7, q = 80.00 and r = 15.900 are rounded off with corresponding percentage errors of 0.5, 0.05 and 0.05. Find the relative error in $\left(\frac{q}{r} - p\right)$

7.(a) Use the trapezium rule with five sub-intervals to estimate: $\int_0^1 5^{2x}\, dx$. Give your answer correct to 3 decimal place.
(b) Determine the exact value of $\int_0^1 5^{2x}\, dx$ to 3 decimal places.
(c) Find the percentage error in the two calculations in (a) and (b) above.

8. Given that f(0.9) = 0.2661, f(1.0) = 0.2420 and f(1.1.) = 0.2179, estimate:
(i) f(0.96)
(ii) $f^{-1}(0.2372)$ using linear interpolation or extrapolation

9. Using a Trapezium rule with six ordinates estimate the value of $\int_0^{\pi} x^2 e^x \, dx$ *to* 2 *d.p*

10. a) Given the equation $ax^2 + bx + c = 0$, show that the Newton Raphson method for solving the above equation is given by

$x_{n+1} = \frac{ax_n^{\,2} - c}{2ax_n + b}$, n = 0, 1, 2, …

b)Show that the equation $3x^3 + x - 4 = 0$ has a root between 0 and 2. Hence using Newton Raphson's method, find the root correct to three decimal

11. a) Two decimal number X and Y are rounded to give x and y with errors $\boldsymbol{e_1}$ and $\boldsymbol{e_2}$ respectively. Show that maximum absolute relative error committed when x^2y is used instead of X^2Y is given by $2\left|\frac{e_1}{x}\right| + \left|\frac{e_2}{y}\right|$

Determine the limits within which the exact value of $\frac{6.042 - 3.21}{12.1}$ lie

12. The bus charges over distances of 4 km and 10 km from town P are shs 2800 and shs 5800 respectively. Estimate the:

(i) charge that is worth 7 km from P.

(ii) distance from P that is worth shs 5300.

13. (a)Use the graphical method to show that the equation $\tan x - 4x = 0$ has a root between 1 and 1.5.

(b) Show that the equation in (a) above yields Newton Raphson's formula $x_{n+1} = \frac{x_n \tan^2 x_n - \tan x_n + x_n}{\tan^2 x_n - 3}$. Hence find the root of the equation correct to three decimal places.

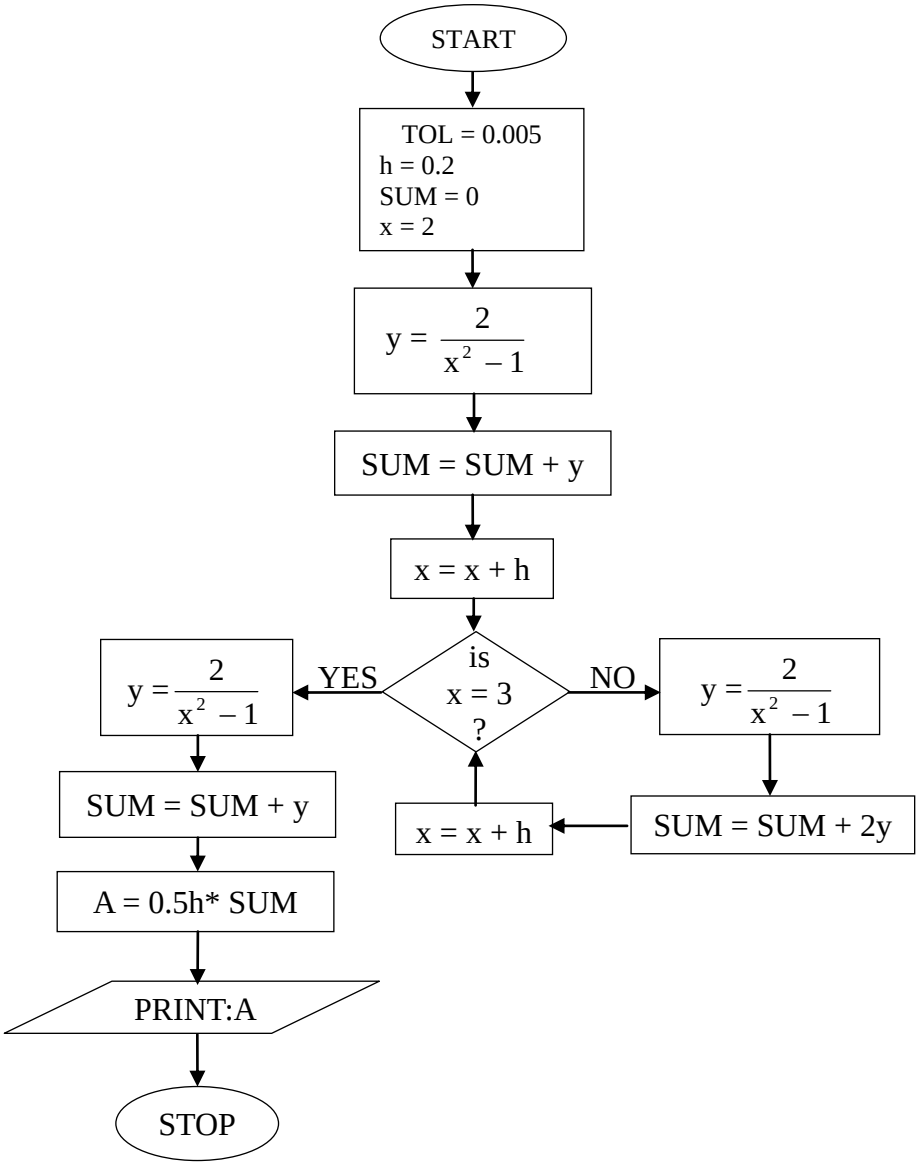
14.(a)Use the trapezium rule with six ordinates to estimate $\int_1^3 \sin^2 x \, dx$ correct to three decimal places.

(b) Find the exact value of $\int_1^3 \sin^2 x \, dx$.

(c) Find the percentage error in your estimation in (a) above and suggest how it can be reduced.

15. By drawing a suitable graph, determine the positive root of the equation $2x^2 - x - 5 = 0$.

16. Study the flow diagram below.



Perform the dry run of the flow chart and state its purpose.

16. (a) Given that the exact quantities A and B are approximated using a and b respectively with corresponding errors ΔA and ΔB, show that the maximum possible error in $\frac{a}{b}$ is $\left|\frac{a}{b}\right|\left(\left|\frac{\Delta A}{a}\right|+\left|\frac{\Delta B}{b}\right|\right)$.

(b) The quantities x = 14.57, y = 17.9112 and z = 4.3 are each rounded off to the given number of decimal places. If d = $\frac{x - y}{y - z}$, find

(i) range of values within which d lies correct to four decimal places,

(ii) percentage error in d correct to two significant figures.

17. Derive the simplest iterative formula based on Newton Raphson method that can be used to find the cube root of a number N.

(a) By drawing a suitable graph, find the value of $\sqrt[3]{12}$.

(b) Using the value from the graph as an initial approximation, find $\sqrt[3]{12}$ correct to three decimal places.

18. Points A and B are 22.5 km apart. A body is timed as it moves from A to B and it takes 0.426 hours. Determine the maximum possible error made in calculating the speed of the body.

19. (a) Show graphically that the equation $2e^x + x^2 - 4 = 0$ has two real roots. Hence obtain the negative root from your graph to 1 d.p.
(b). Use Newton Raphson's method to find the negative root of the equation, correct to three significant figures.

20. The table below is an extract from a table of tangents.

θ	24'	30'	36'	42'
$\tan 25^0\theta'$	0.4748	0.4770	0.4791	0.4813

Use linear interpolation or extrapolation to find;

(i) $\tan 25^0 18'$

(ii) $\tan^{-1}(0.4775)$

End of Numerical methods

3. MECHANICS

1. Projectile is fired from the ground with initial velocity $3\boldsymbol{i} + 4\boldsymbol{j}\ ms^{-1}$. Find its:
(i). position vector at any time .
(ii). horizontal range.

2. Particles of masses 3 g, 5 g, 2 g and 4 g act at points (1, 6) ,(1, 5), (2, -3) and (-1, -4) respectively in the plane. Find the coordinates of their centre of gravity and its distance from the origin .

3. Five forces 10N, 3N, 5N, 6N and √3N act along sides AD, DC, CB, BA and AC of an isosceles trapezium, where AD=DC=CB=1m and AB=2m, their directions being indicated by order of the letters.
(a). Find the magnitude and direction of the resultant of the forces.
(b). Calculate the distance from A, where the line of action of the resultant cuts AB.

4. A particle of mass 0.5 kg is acted upon by an accelerating force of $(18t^2\boldsymbol{i} - 9t\boldsymbol{j})$and passes through point (-4, 6) with a speed of $(4.5\boldsymbol{i} - 6\boldsymbol{j})ms^{-1}$.
(a). Find the velocity and position vector of the particle t seconds after passing through the point .
(b). Calculate the power developed at t=2 s .

5. (a). To a person on a ship A moving due east at 15kmh^{-1}, ship B appears to be moving at 12kmh^{-1} in the direction $N30^0E$. Find the true velocity of ship B.
(b). Two ships A and B have the following position vectors, r , and velocity vectors, v , at the times stated.
$rA = (-2\boldsymbol{i} + 3\boldsymbol{j})km, VA = (12\boldsymbol{i} - 4\boldsymbol{j})kmh^{-1}\ at\ 11{:}45\ am$
$rB = (8\boldsymbol{i} + 7\boldsymbol{j})km, VB = (2\boldsymbol{i} - 14\boldsymbol{j})kmh^{-1}\ at\ 12{:}00\ noon$
Assuming that the ships do not alter their velocities, find the least distance between A and B hence the bearing of A from B when they are nearest each other.

6.Forces of $\begin{pmatrix}-2\\-3\end{pmatrix}N, \begin{pmatrix}7\\4\end{pmatrix}N, \begin{pmatrix}P\\2\end{pmatrix}N$ and $\begin{pmatrix}1\\-Q\end{pmatrix}N$ are in equilibrium. Find the value of P and Q

7. Three particles whose masses are m, m and M are fixed to points (0,0), (2,0) and (1,4) respectively. If the centre of mass of the particles is at (1, 1), show that. 2m =3M.

8. To a passenger in a bus moving at 80kmh^{-1} in the direction $N40^0W$, a plane appears to fly from the west at 30kmh^{-1}. Find the true speed and course of the plane

9. A point O is vertically above a fixed point A on a horizontal plane. A particle P is projected from O with a speed 5V ms^{-1} at an angle $\cos^{-1}\left(\frac{3}{5}\right)$ above the horizontal, and hits the plane at a point B, at a distance $\frac{48V^2}{g}$ metres from A.
(i) Find the distance of P from O when it is directly level with it.
(ii) Show that the height of O above A is $\frac{64V^2}{g}$ metres.

10. A particle P of weight 4,9N rests on a long horizontal table. It is connected by a light inelastic string, passing over a smooth light pulley at the edge of the table; to another particle B of mass 2kg hanging freely. The system is released from rest, and after 2seconds P collides and coalesces with a stationary particle of mass 0.1kg at rest on the table. The coefficient of friction between the table and the weight is 0.25.
(a) calculate the:
(i) acceleration of the system just before collision.
(ii) tension in the string just after collision.
(b) Find the change in kinetic energy of P immediately after collision.

11. A car approaching a town does two successive half-kilometers in 16 and 20 seconds respectively. Assuming the retardation is uniform. Find the further distance the car runs before stopping.

12. Two forces of magnitude 8N and PN are inclined at an angle 60^o. Find P if the resultant is of magnitude 17N

13. To a motor cyclist travelling due North at 50kmh^{-1}, the wind appears to come from North West at 60kmh^{-1}. Find the true velocity of the wind

14. Forces of magnitude 3N, 4N,4N, 3N and 5N act along the lines AB, BC, CD,DA and AC respectively of the square ABCD whose side has length a units. The directions of the forces are indicated by the order of the letters.
(a) Find the magnitude and direction of the resultant force.
(b) If the line of action of the resultant force cuts AB produced at E, find the length AE.

15. a) The position vector of ship P relative to Q at any time t is given by ${}_{\mathrm{P}}\mathbf{r}_{\mathrm{Q}} = \begin{pmatrix} 4 \\ 13 \end{pmatrix} + \begin{pmatrix} -2 \\ -4 \end{pmatrix} t$ metres. Find the:

i) time when they are nearest together

ii) closest distance between the two ships

b) Projectile is fired from the ground with initial velocity $3\boldsymbol{i} + 4\boldsymbol{j}$ ms^{-1}. Find its ; (i) Position vector at any time t (ii) Horizontal range

16. A body of mass mkg lies on a rough plane inclined at θ^0 to the horizontal. When a force of $\frac{mg}{2} N$ parallel to and up the plane is applied to the body, it is just about to move up the plane. When a force of $\frac{mg}{4} N$ parallel to and down the plane is applied to the body, it just about to move down the plane. Calculate correct to two decimal places the value of:

(i) θ

(ii) The coefficient of friction between the body and the plane.

17. A car of mass 2000 kg has a maximum power of 45 kW up a slope of 1 in 70 against a constant resistance to motion of 300 N. Find the

(i) driving force of the car engine,

(ii) acceleration of the car up the slope when its speed is 50 ms^{-1}.

18. Particles of masses 4 kg, 3 kg, 2 kg and 6 kg respectively, act at the corners of a rectangle ABCD in which AB = 18 cm and AD = 12 cm. Find the position of the centre of gravity of the whole system.

19. An aircraft which can travel at $100kmh^{-1}$ in still air, is to be flown from town P to town Q, 340 km apart with Q on the bearing of 060^0 from P. If there is a steady wind of $40kmh^{-1}$ blowing from the west, find the time taken for the flight.

20. (a)A particle is projected from a point O with speed u at an angle of elevation θ to the horizontal. Show that its height y above O when it has travelled a distance x horizontally is given by $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$.

(b)Find the velocity and angle of projection of a particle which passes in a horizontal direction just over the top of a wall which is 12 m high at a horizontal distance of 32 m from the point of projection.

21. (a)If the acceleration of a particle moving with simple harmonic motion is given by $a = -\omega^2 x$, where ω is a constant and x is the distance from the centre of motion, show that $x = A\sin(\omega t + \varepsilon)$, where ε is a constant and A is the amplitude.

(b)A particle moving with S.H.M has velocities of $7.5ms^{-1}$ and $4ms^{-1}$ when at distance of $0.9m$ and $0.2m$ respectively from the end points of its path. Find the length of its path.

22. (a) ABCD is a square. Forces of magnitudes 9 N, 5 N and $3\sqrt{2}$ N act along $\overrightarrow{AB}$, $\overrightarrow{BC}$ and $\overrightarrow{BD}$ respectively. Find the magnitude and direction of their resultant.

(b) A non uniform ladder AB whose centre of gravity is 2 m from end A is of length 6 m and weight W. The ladder is inclined at angle of θ to the vertical with its end B against a rough vertical wall and end A on a rough horizontal ground with which the coefficients of friction at each point of contact is μ. If the ladder is about to slip when a man of weight 5W ascends two–thirds of the way up the ladder, show that $\tan\theta = \frac{18\mu}{11-7\mu^2}$.

23. Particles of masses 4kg, 3kg, 2kg, 9kg and 2kg are placed at points with position vectors: 2**i** + 4**j**, -3**i** – 2**j**, **i** – 4**j**, -3**i** + 5**j** and 4**i** + 3**j** respectively. Find the position vector of centre of gravity of the system of particles.

24. A car travelling along a straight road covers consecutive distances 1km and 2km in equal time interval of 10 minutes. Find the

(i) acceleration of the car,

(ii) initial velocity.

25. A bullet of mass 100 grams travelling at 400ms^{-1} horizontally is fired into a block of mass 20kg at rest on a horizontal table. If the coefficient of friction between the table and the block is 0.2, calculate the;

(i) acceleration of the block if the bullet is embedded in the wood,

(ii) distance moved by the block.

26. (a) A particle performs simple harmonic motion between two points A and B which are 4m apart with point O as center. When the particle is at point 1 m from O, its speed is $\frac{1}{\sqrt{3}}$ ms^{-1}.

Calculate the

(i) acceleration when the particle is 0.4m from A,

(ii) displacement of the particle after 2seconds.

(b) A particle of mass 2kg is freely suspended from the end B of a light elastic string of natural length 1m and the other end of the strong is fixed to a point A. When in equilibrium at a point E, AE = 1.5m. The particle is then pulled vertically downwards to a point C, 0.2m from E and released.

(i) Show that the subsequent motion is simple harmonic.

(ii) Find the time taken by the particle to reach a point 0.15m from C for the first time.

27. The points A, B, C and D are the vertices of a square of side 4m. Forces of magnitudes 8N, 3N, $4\sqrt{2}$ N, 6N and 3N act along the sides BA, CB, AC, DA and DC respectively, the direction of the forces being indicated by the order of the letters. Find the

(a) magnitude of the resultant force.

(b) equation of line of action of the resultant force taking AB and AD as the horizontal and vertical axes respectively.

(c) distance from A where the line of action of the resultant crosses AB.

28.(a) A car of mass 1500kg travelling along a horizontal road at a maximum power has a maximum speed of 150kmh^{-1}. If there is a resistance of 60N, find the

(i) maximum power,

(ii) acceleration when the car is travelling at 72kmh^{-1} if power remains constant.

(b) A hammer of mass 4.5kg falls through a vertical height of 1m and hits a nail of mass 50 grams directly without rebounding. If the nail is driven into a piece of wood a depth of 2cm, find average resistance to penetration assuming that it is constant

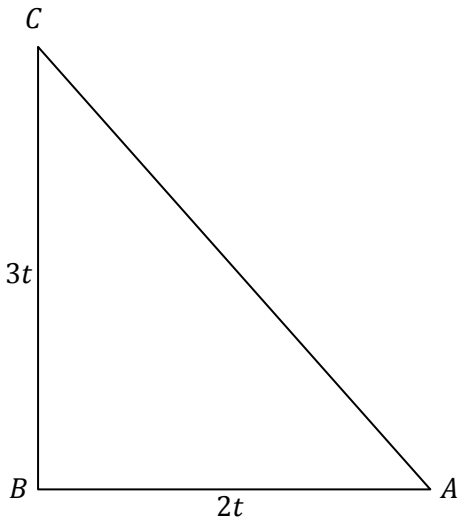
29. At 12:00 noon, the position vector of particle A relative to particle B is $2\boldsymbol{i} + 20\boldsymbol{j} + 18\boldsymbol{k}$ kilometres and velocities of A and B by then are $4\boldsymbol{i} + \boldsymbol{j} - 2\boldsymbol{k}\ kmh^{-1}$and $6\boldsymbol{i} + 3\boldsymbol{k}\ kmh^{-1}$respectively. Find the:

(i). time when they are nearest to each other.

(ii). Closest distance between them.

(b). To a girl running at 6ms^{-1} on a bearing of 155^0, a low flying bird appears to be moving at 7ms^{-1} on a bearing of 250^0 . Find the true velocity of the bird.

30. ABCD is a uniform triangular lamina right angled at B where AB= 2t and BC=3t.



The midpoint P and Q of BC and CA respectively are joined and the portion PQC cut off.

(a) Find in terms of t, the distance from AB and BC of the centre of gravity of the lamina ABPQ.

(b) When the lamina ABPQ is suspended freely from vertex A, find the angle AB makes with the vertical.

End of Mechanics